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The influence of the diameter-to-thickness ratio on the stability of circular tubes under cyclic bending

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Abstract

This paper presents the experimental results on the influence of the diameter-to-thickness ratio (D/t ratio) to the response and stability of circular tubes subjected to symmetrical cyclic bending. To highlight the influence of the D/t ratio to the response and stability of circular tubes under symmetrical cyclic bending, the raw tubes were slightly machined on the outside surface to obtain the desired D/t ratio. However, the magnitudes of the inside diameter were intact and the same for all tested specimens. It was observed that if a certain amount of controlled curvature is considered, specimens with smaller outside diameters have a few number of cycles to produce buckling than these with larger outside diameters. In addition, although four groups of tested specimens had four different D/t ratios, four parallel straight lines can be seen from the relationship between the controlled curvature and the number of cycles to produce buckling in log–log scale.

Finally, the empirical relationship, proposed by Kyriakides and Shaw (1987), was modified so that it can be used for simulating the relationship between the controlled curvature and the number of cycles to produce buckling for circular tubes with different D/t ratios. The simulation was compared with the experimental test data. Good agreement between the experimental result and modified empirical relationship has been achieved. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Circular tube; Symmetrical cyclic bending; Stability; Curvature; Ovalization

1. Introduction

Circular tubes are often employed in many industrial applications, such as offshore pipelines, platforms in offshore deep water, heat exchangers in power plants and nuclear reactors, etc. The bending of a circular tube can lead to the ovalization of the tube cross-section. The reverse bending and subsequent repeated cyclic bending cause a gradual growth of ovalization. The growth of ovalization causes a progressive reduction in the bending rigidity of the tube. Once a critical value of ovalization is reached, the tube will buckle. Therefore, understanding of the response and stability of circular tubes under bending is of importance in many industrial applications.

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Since 1980, Kyriakides and his coworkers have studied circular tubes subjected to montonic or cyclic bending with or without external pressure. Kyriakides and Shaw (1982) investigated the response and stability of elastoplastic pipes under combined bending and external pressure. They determined the maximum moment and curvature, as a function of the material and geometric parameters for different pressures. Shaw and Kyriakides (1985) studied the inelastic behavior of circular tubes subjected to cyclic bending. They demonstrated that reverse bending and subsequent repeated cyclic bending cause a gradual growth of the ovalization of the tube cross-section. Kyriakides and Shaw (1987) extended the analysis to the stability of circular tubes subjected to cyclic bending. Their results indicated that under curvature-symmetrical loading, a tube progressively ovalizes to a critical value at which it buckles. The ovalization's critical value was shown to be approximately equal to the value obtained just prior to the buckling under monotonic bending. Corona and Kyriakides (1988) investigated the stability of circular tubes subjected to combined bending and external pressure. The curvature–pressure interaction collapse envelopes were generated for two different loading paths involving bending followed by pressure and pressure followed by bending. According to their investigation, the loading path strongly influences the shape of the envelopes in the curvature–pressure interaction. Corona and Kyriakides (1991) studied the degradation and buckling of circular tubes under cyclic bending and external pressure. They established the effect of the cyclic bending history and the external pressure on the rate of ovalization accumulation and the onset of instability. Kyriakides and Ju (1992a,b) studied experimentally and theoretically the bifurcation and localization instabilities on 6061-T6 aluminum tubes under pure bending. They discovered that the thinner shells developed short wavelength periodic ripples on the compressive side of the shell and buckled soon after the ripples appeared. However, thicker shells exhibited a limit load instability as a direct consequence of the ovalization of the shell cross-section caused by bending.

In recent years, Pan and his coworkers experimentally and theoretically investigated the response and stability of circular tubes subjected to monotonic or cyclic bending. Pan and Leu (1997) used the endochronic theory to investigate the collapse of thin-walled tubes subjected to bending. The experimental data from 6061-T6 aluminum and 1018 steel tubes under cyclic bending tested by Kyriakides and Shaw (1987) were compared with the theoretical simulation. Pan et al. (1998) designed a new measurement apparatus, which can be placed at the mid-span of a circular tube specimen and is suitable for simultaneous experimental determinations of the tube curvature and ovalization of the tube cross-section. For testing the newly designed apparatus, a tubular specimen of 304 stainless steel was cyclically bent. Pan and Fan (1998) investigated the effect of the prior curvature rate at the preloading stage on the subsequent creep (hold constant moment for a period of time) or relaxation (hold constant curvature for a period of time) behavior. Thin-walled tubes of 304 stainless steel were used in this investigation. It has been found that the curvature rate at the preloading stage has a strong influence on the subsequent creep or relaxation deformation under pure bending. Furthermore, Pan and Her (1998) used the aforementioned new apparatus to investigate the response and stability of 304 stainless steel tubes subjected to cyclic bending with different curvature rates. They found that the higher the applied curvature rate, the greater the degree of hardening of the metal tube. However, the ovalization of tube cross-section increased when the applied curvature rate increased. Pan and Hsu (1999) experimentally and theoretically studied the viscoplastic behavior of 304 stainless steel tubes subjected to cyclic bending. The endochronic viscoplastic theory, which was proposed by Pan and Chern (1997), was used to investigate the viscoplastic behavior of the tubes under cyclic bending.

In this study, the influence of the D/t ratio on the stability of circular tubes subjected to symmetric cyclic bending was investigated. A four-point bending machine (facilities reported in previous studies (Pan et al., 1998; Pan and Fan, 1998; Pan and Her, 1998; Pan and Hsu, 1999)) was used for conducting the present tests. The circular tube material chosen for this study was 304 stainless steel. The curvature-ovalization measurement apparatus (COMA), designed and reported previously by Pan et al. (1998), was used for conducting the curvature-controlled cyclic bending test. The magnitudes of curvature and ovalization of the tube cross-section were measured simultaneously. The magnitude of bending moment was obtained from the two load cells mounted in the bending device. To investigate the influence of the *D*/*t* ratio on the stability of circular tubes under cyclic bending, the tubes were slightly machined on the outside surface to obtain the desired *D*/*t* ratio. The inner diameter for all tested specimens was left intact. It was observed from the experimental moment–curvature curve that the 304 stainless steel tube cyclically hardens and becomes gradually steady after a few cycles for symmetrical curvature-controlled cyclic bending. It was also found from the experimental ovalization–curvature curve that the ovalization of the tube cross-section increased in a ratcheting manner with the number of cycles. Due to the progressive accumulation of tube cross-section ovalization during the cyclic bending, the tube buckles eventually. In addition, for a certain amount of curvature-controlled bending, specimens with a smaller outside diameter have fewer number of cycles before buckling than those with a larger outside diameter. Although four groups of tested specimens had four different *D*/*t* ratios, four parallel straight lines can be seen from the relationship between the controlled curvature and the number of cycles to produce buckling in log–log scale.

Finally, the empirical relationship of Kyriakides and Shaw (1987) was modified so that it could be used for simulating the relationship between the controlled curvature and the number of cycles to produce buckling for circular tubes with different *D*/*t* ratios. It has been shown from the comparison between the simulation of the modified relationship and the experimental data that the modified empirical formulation can properly simulate the experimental result.

2. Experimental facilities

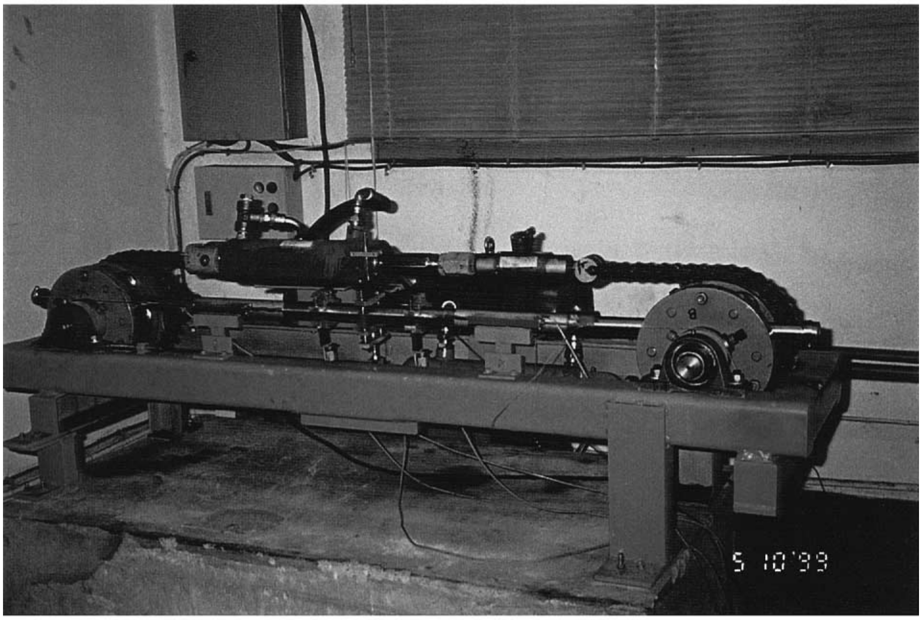
The experiments were conducted using a test facility designed by Pan and his coworkers, consisting of a pure bending device and a COMA. The bending device was used for conducting the cyclic bending tests and the COMA was used for measuring the variations in tube curvature and the ovalization of the tube cross-section.

The bending device is shown in Fig. 1(a) and (b). It was designed as a four-point bending machine capable of applying bending and reverse bending (Pan et al., 1998; Pan and Fan, 1998; Pan and Hsu, 1999; Pan and Her, 1998). This bending device is similar to the bending devices reported by Kyriakides and coworkers. The bending device consists of two heavy assemblies resting on two beams. Heavy chains run around these sprockets and are connected to two hydraulic cylinders and load cells forming a closed loop. Each tube tested is fitted with solid rod extensions. The rods are chamfered at the ends to reduce stress concentrations and to avoid premature buckling at the end of the test specimen. The test specimen assembly is engaged by the bending device through four rollers located on each sprocket.

Bending is achieved by contracting either of the cylinders in the process causing the rotation of the sprockets. As the sprocket rotates, the test specimen assembly is loaded by a couple formed by concentrated loads from two of the rollers. During bending, the rolling contact between the test specimen assembly and the device guarantees the freedom of movement of the tube in the axial direction. Bending in the reverse sense is achieved by reversing the direction of the flow in the hydraulic circuit. The applied bending moment is directly proportional to the tension in the chain, which is monitored by two load cells in the chain loop.

The magnitudes of the tube curvature and the ovalization of the tube cross-section were measured by a special instrument COMA, which was designed by Pan et al. (1998). Fig. 2(a) and (b) shows a picture and a schematic drawing of the COMA, respectively. The apparatus is a lightweight instrument, which can be mounted close to the tube's mid-span during the test. By using a magnetic detector (middle part of the COMA), it can monitor the changes in the major and minor diameters of the tube cross-section (the ovalization of tube cross-section). Simultaneously, it can measure variations in the tube curvature close to the mid-span from the inclinometer signals. Based on the fixed distance between the two-side inclinometers and

(a)



(b)

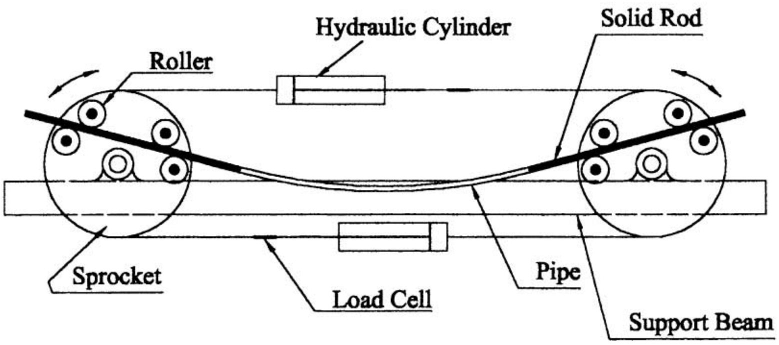


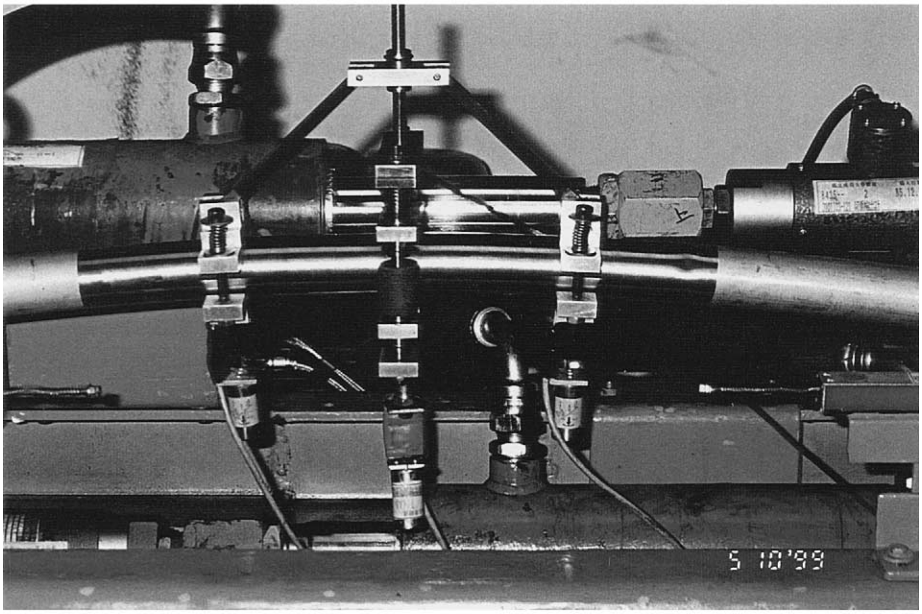
Fig. 1. (a) A picture of the bending device and (b) a schematic drawing of the bending device.

the angle changes detected by the two-side inclinometers, the tube curvature can be obtained (see Fig. 2(b)). A detailed description of the COMA can be found in the previous work by Pan et al. (1998).

3. Material and experimental procedure

The experiments were carried out using hot-rolled SUS 304 stainless steel circular tubes with the chemical composition of Cr 18.36, Ni 8.43, Mn 1.81, Si 0.39, C 0.05, P 0.28, S 0.04, and the remainder Fe. The yield stress is 205 MPa, the tensile ultimate stress is 520 MPa and the percent elongation is 35%. The test specimens had a nominal outside diameter D of 38.1 mm and a wall thickness t of 1.5 mm ($D/t = 25.4$)

(a)



(b)

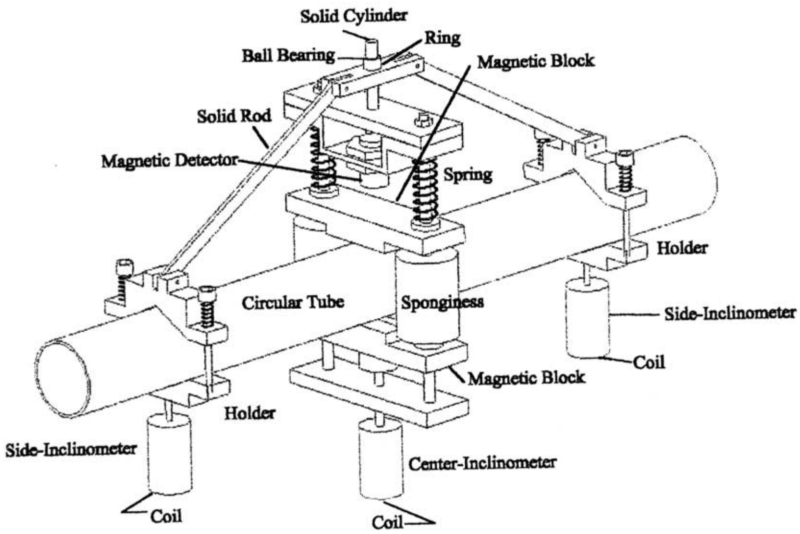


Fig. 2. (a) A picture of the COMA and (b) a schematic drawing of the COMA.

originally. To obtain the desired D/t ratio, the raw tested specimens were slightly machined on the outside surface. The magnitudes of the D/t ratio were selected to be 30, 40, 50 and 60 in this study. Therefore, the outside diameters of the raw specimens $D = 38.1$ mm were machined to obtain the outside diameters of 37.60, 36.95, 36.56 and 36.31 mm, respectively. However, the inside diameters were intact and the same magnitude for all tested specimens (inside diameter = 35.1 mm). Fig. 3 shows the dimensions of four different D/t ratios of the tested specimens.

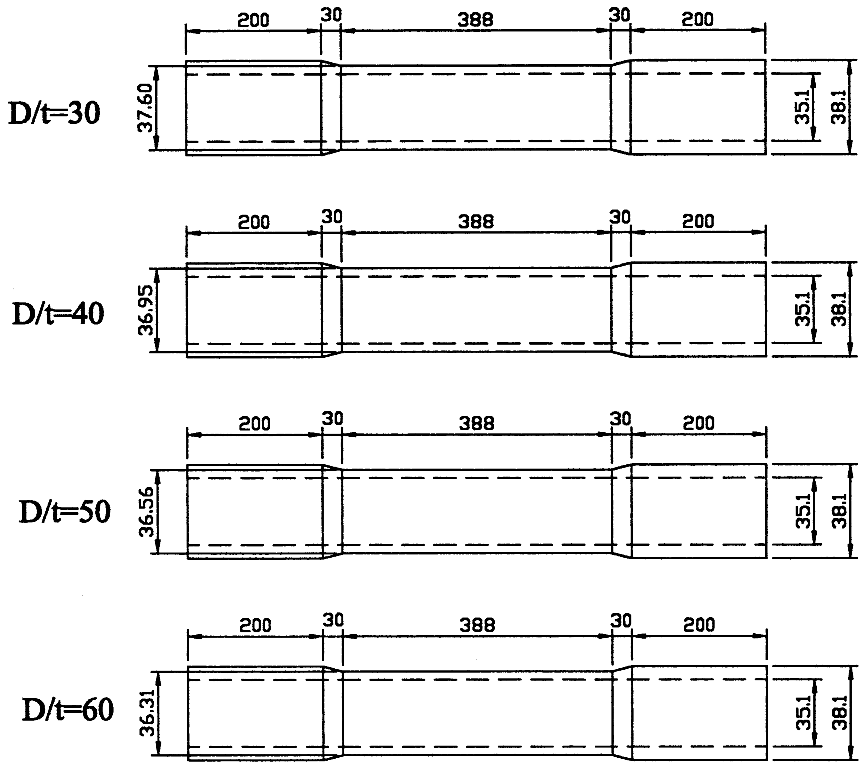


Fig. 3. Geometry of the tested circular tubes (all dimensions in mm).

In this study, the elasto-plastic response and collapse of SUS 304 stainless steel circular tubes subjected to cyclic bending were investigated. The test was a curvature-controlled cyclic bending test with the curvature amplitude from ± 0.1 to $\pm 0.65\ \mathrm{m}^{-1}$. A large number of experiments were conducted for the curvature-symmetrical case in order to carefully establish the response characteristics and the circular tube instability. During the test, the magnitude of the bending moment was measured by two load cells mounted in the bending device, the magnitudes of the curvature and the ovalization of the tube cross-section were measured by the COMA. In addition, the number of cycles to produce buckling were also recorded.

4. Experimental results

Fig. 4(a)–(d) presents the experimental results of the moment (M)–curvature (κ) curves for circular tubes with the D/t ratios of 30, 40, 50 and 60, respectively. It is shown that the 304 stainless tube was cyclically hardened for symmetrical curvature cyclic bending and the moment–curvature loops became gradually steady after a few cycles. Due to the thinner thickness of the circular tube with a higher D/t ratio, a lower magnitude of the bending moment was needed to bend the circular tube into the desired curvature. Fig. 5(a)–(d) depicts the corresponding experimental results of ovalization ($\Delta D/D$)–curvature (κ) curves for circular tubes with the D/t ratios of 30, 40, 50 and 60, respectively. The ovalization of tube cross-section is defined as $\Delta D/D$, where D is the outside diameter and ΔD is the change in outside diameter. It was found that one of the main characteristics of this phenomenon was the gradual accumulation of ovalization in the

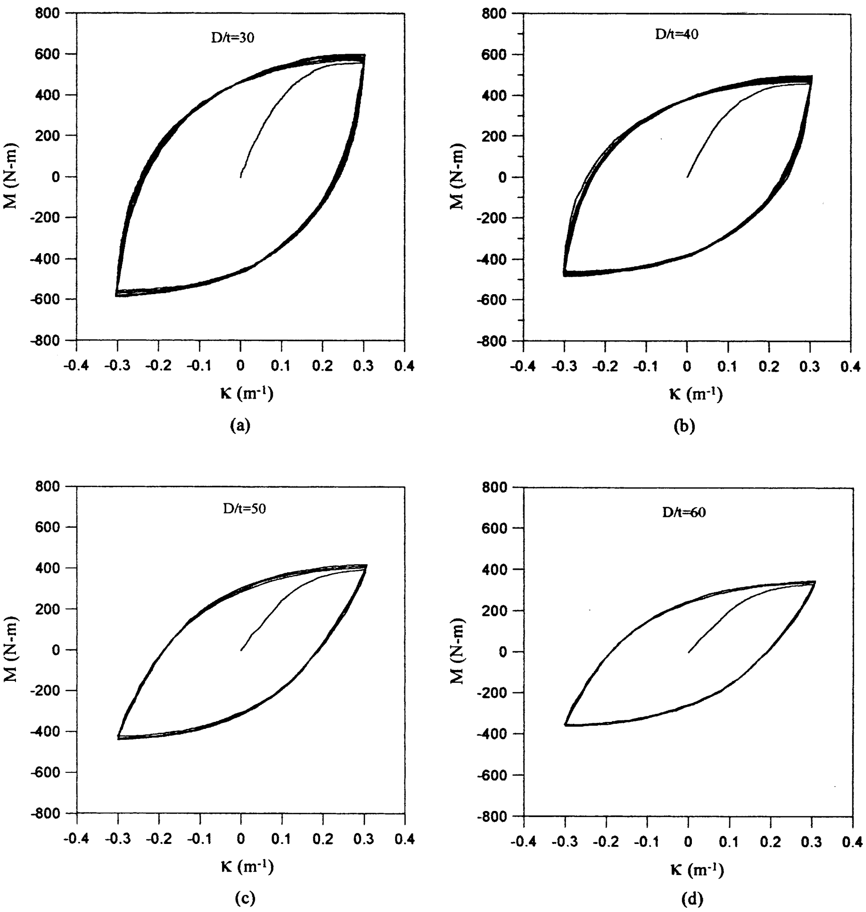


Fig. 4. Typical experimental moment (M)–curvature (κ) curves for D/t: (a) 30, (b) 40, (c) 50, and (d) 60.

tube cross-section, induced by cyclically bending the tube into the plastic range. Persistent cycling eventually leads to buckling. This phenomenon was also discovered by Kyriakides and Shaw (1987), who tested 6061-T6 aluminum and 1018 steel tubes. It was also found that the higher the D/t ratio for a circular tube, the faster the accumulation of ovalization in the tube cross-section. Fig. 6(a)–(d) demonstrates some experimental results of the variations in ovalization ($\Delta D/D$), at two extreme curvature values for each cycle, with the number of cycles (N) for circular tubes with the D/t ratios of 30, 40, 50 and 60, respectively. Due to the rapid increase in tube ovalization for higher controlled curvature, the number of cycles required for buckling to occur was reduced. In addition, the maximum ovalization reached for each case was approximately the same. A similar result was also found for 6061-T6 aluminum and 1018 steel tubes tested by Kyriakides and Shaw (1987) and 304 stainless steel tubes tested by Pan and Her (1998). However, the magnitude of the maximum ovalization reached was higher for circular tubes with a higher D/t ratio than those with lower D/t ratio (see the dot lines Fig. 6(a)–(d)). Finally, the magnitude of controlled cyclic curvature range (κ_c) versus the number of cycles to produce buckling (N_b) curves for circular tubes with the D/t ratios of 30, 40, 50 and 60 are shown in Fig. 7. It was found that a higher curvature leads to a shorter number of cycles to produce buckling. A similar result was also found for 6061-T6 aluminum and 1018 steel tubes tested by Kyriakides and Shaw (1987).

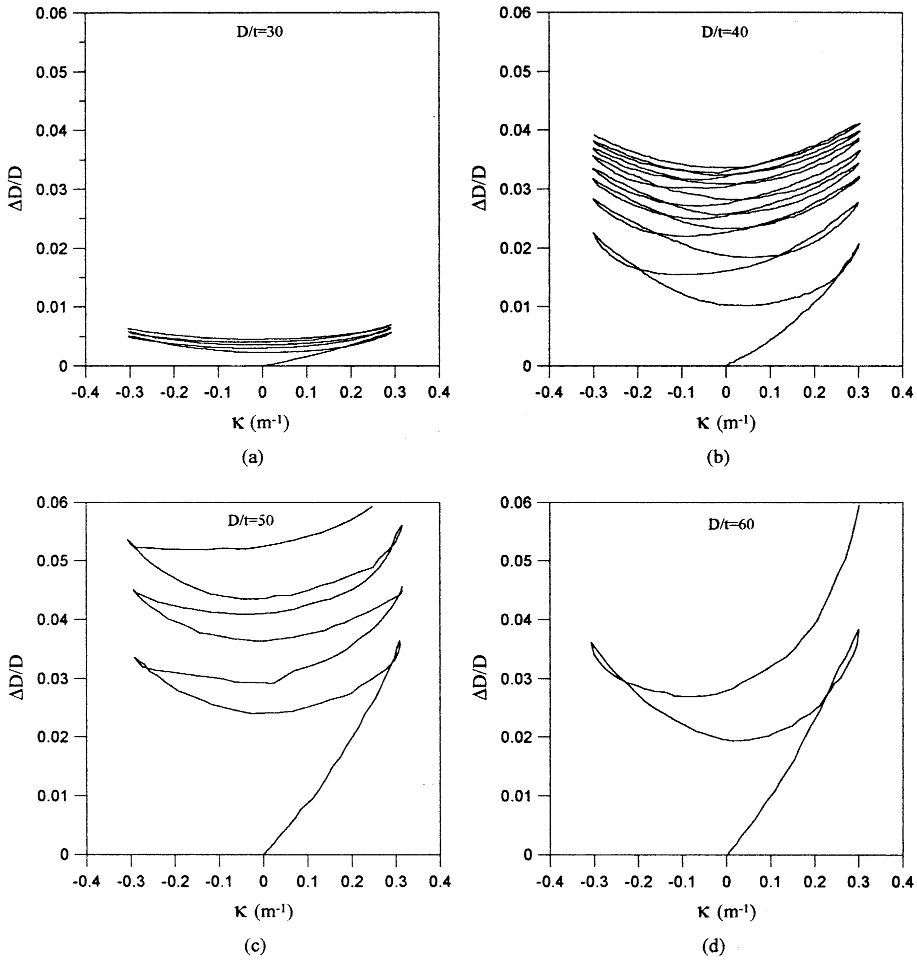


Fig. 5. Corresponding ovalization ($\Delta D/D$)–curvature (κ) curves for D/t: (a) 30, (b) 40, (c) 50, and (d) 60.

5. Discussion

Kyriakides and Shaw (1987) proposed an empirical relationship between the magnitude of controlled cyclic curvature range (κ_c) and the number of cycles to produce buckling (N_b) as

$$\kappa_c = A(N_b)^{-\alpha}, \tag{1}$$

where A and α are material parameters, which are related to the material properties and the D/t ratio. The material parameter A is the controlled cyclic curvature magnitude at $N_b = 1$, and α is the slope in the log–log plot. Based on the experimental data of circular tubes with $D = 31.75$ mm and $t = 0.889$ mm ($D/t = 35.7$), reported by Kyriakides and Shaw (1987), the magnitudes of A and α were calculated to be: $A = 0.968$ m^{-1} and $\alpha = 0.078$ for 1018 steel and $A = 0.988$ m^{-1} and $\alpha = 0.078$ or 0.12 for 6061-T6 aluminum.

For simulating the viscoplastic collapse of circular tubes, Pan and Her (1998) modified Eq. (1) to be

$$\kappa_c = B(N_b)^{-\alpha}, \tag{2}$$

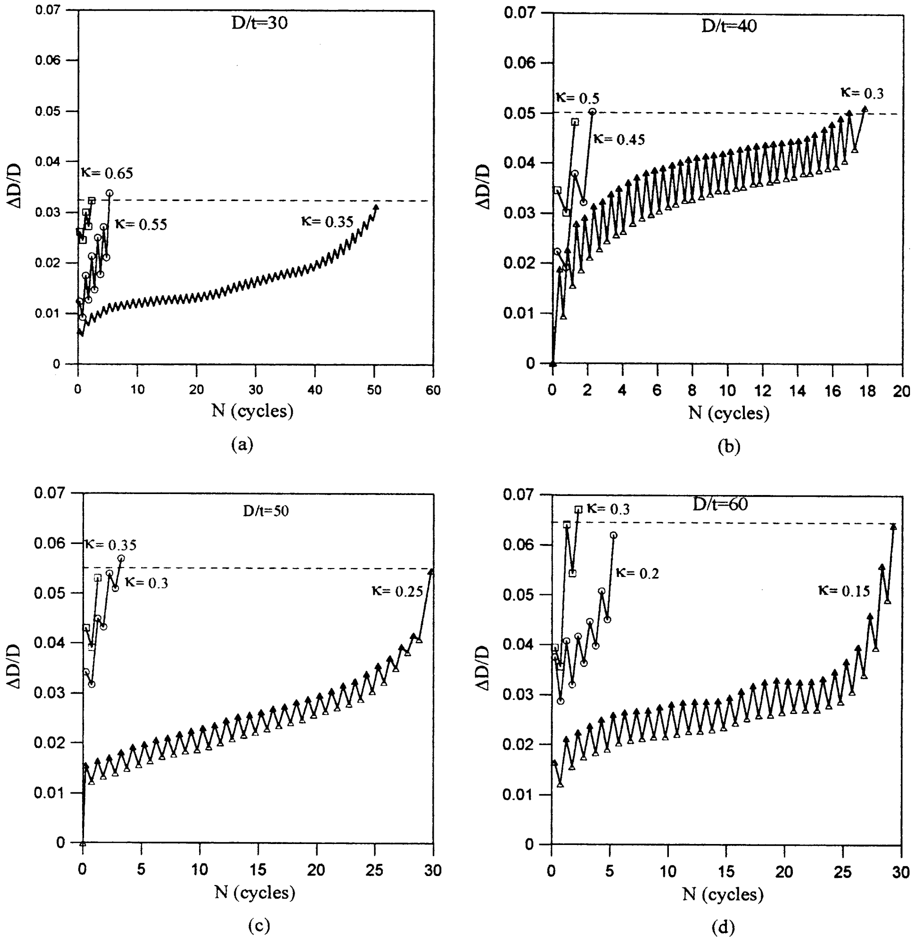


Fig. 6. Typical experimental data of the ovalization ($\Delta D/D$) at extreme curvatures versus number of cycles (N) for D/t: (a) 30, (b) 40, (c) 50, and (d) 60.

where B is a function of the controlled-curvature rate, and can be expressed as

$$B = B_0 + \beta \left[\log \frac{\dot{\kappa}_c}{\dot{\kappa}_0} \right]^2, \tag{3}$$

where B_0 is a material parameter for the lowest controlled-curvature rate and $\dot{\kappa}_0$ is the lowest controlled cyclic curvature rate, $\dot{\kappa}_c$ is the other controlled cyclic curvature rate and β is the material parameter. For circular tubes of 304 stainless steel with D/t ratio of 50, the magnitudes of B_0 and α were determined to be 0.357 m^{-1} and 0.118, respectively by letting $\dot{\kappa}_c = \dot{\kappa}_0$. Based on the variation of κ_c–N_b curves for different controlled-curvature rates, the value of β was found to be 0.017 in their study (Pan and Her, 1998).

In this study, the κ_c versus N_b curves for circular tubes with the D/t ratios of 30, 40, 50 and 60 are shown in Fig. 7. The corresponding experimental results plotted in log–log scale are shown in Fig. 8. Four straight dot lines in this figure, determined by the least-square method, denote four different D/t ratios of tested circular tubes. For a certain D/t ratio of tested circular tubes, the experimental data of κ_c versus N_b (see Fig. 8)

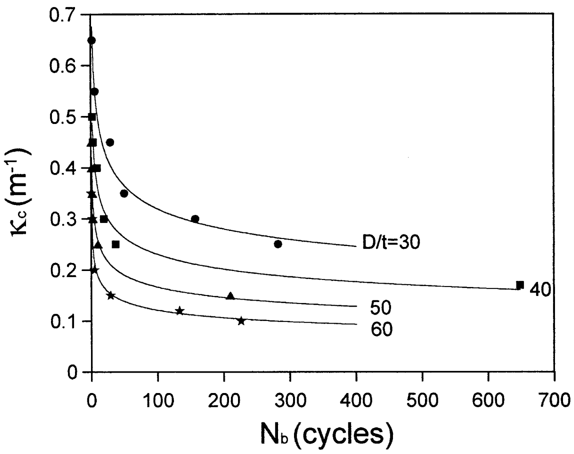


Fig. 7. Controlled cyclic curvature range (κ_c) versus the number of cycles to produce buckling (N_b) curves for circular tubes with four different D/t ratios.

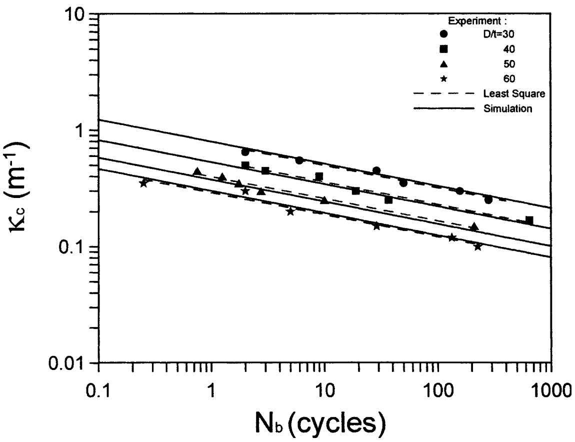


Fig. 8. Controlled cyclic curvature range (κ_c) versus the number of cycles to produce buckling (N_b) curves for circular tubes with four different D/t ratios in log–log scale.

almost fall on a straight dot line. Based on Eq. (1), the material parameter A are determined to be 0.771, 0.557, 0.400 and 0.290 for circular tubes with the D/t ratios of 30, 40, 50 and 60, respectively. The four straight dotted lines are almost parallel to one another. The magnitude of α is approximately equal to 0.19 for four different D/t ratios of circular tubes. Fig. 9 shows the relationship between the material parameter A and the D/t ratio plotted in log–log scale. A straight line can be reasonably determined by the least-square method. Therefore, a formulation for the influence of the D/t ratio on the collapse of the circular tubes is thus proposed to be

$$\kappa_c = C(N_b)^{-\alpha}, \quad (4)$$

where C is a function of D/t ratio, and can be expressed to be

$$C = C_0(D/t)^{-\gamma} \quad (5)$$

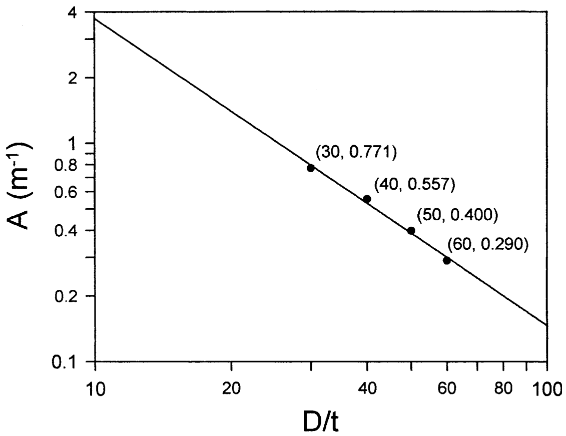


Fig. 9. The material parameter A versus D/t ratio curves in log–log scale.

or

$$\log C = \log C_0 - \gamma \log(D/t),$$

where C_0 and γ are material parameters. In this study, the magnitudes of C_0 and γ can be found in Fig. 9 to be 95.058 m^{-1} and 1.406 for SUS 304 stainless steel tubes. Note that Eqs. (4) and (5) can be used under the circumstances of the constant inner diameter of the circular tubes. The simulated results based on Eqs. (4) and (5) are also demonstrated in Fig. 8 by the solid lines. Good agreement between the experimental and simulated results has been achieved. Fig. 10 shows a picture of the local buckling of some SUS 304 stainless steel tubes under symmetrical cyclic bending.

Fig. 10. A picture of the local buckling for some 304 stainless steel tubes.

6. Conclusions

This study investigated the influence of the D/t ratio on the response and stability of SUS 304 stainless steel circular tubes subjected to cyclic bending. These experiments were carried out under curvature-controlled cyclic bending. The bending device and COMA, designed by Pan et al. (1998), was used for conducting these experiments. Based the experimental and theoretical results, we present the following conclusions:

(1) From the moment–curvature curve, the 304 stainless steel tube exhibits cyclic hardening for symmetrical cyclic bending test. However, the moment–curvature loops become gradually steady after a few cycles. The gradual accumulation of the ovalization of the tube cross-section, which is induced by cyclically bending the tube into the plastic range. Persistent cycling eventually leads to buckling.

(2) The maximum ovalization reached for each D/t ratio was approximately the same. However, the magnitude of the maximum ovalization reached was higher for circular tubes with a higher D/t ratio than that with a lower D/t ratio.

(3) For a certain amount of controlled cyclic curvature range, circular tubes with smaller outside diameters require fewer number of cycles to produce buckling than those with larger outside diameters.

(4) From κ_c versus N_b curves for four different D/t ratios of circular tubes plotted in log–log scale, four straight parallel lines, determined by the least-square method, can be reasonably obtained for four different D/t ratios of circular tubes.

(5) A formulation (Eqs. (4) and (5)) between the κ_c and N_b included the influence of the D/t ratio for circular tubes was proposed in this paper. It is shown that the formulation can adequately simulate the experimental results.

Acknowledgements

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References

Corona, E., Kyriakides, S., 1988. On the collapse of inelastic tubes under combined bending and pressure. Int. J. Solids Struct. 24 (5), 505–535.

Corona, E., Kyriakides, S., 1991. An experimental investigation of the degradation and buckling of circular tubes under cyclic bending and external pressure. Thin-Walled Struct. 12, 229–263.

Kyriakides, S., Ju, G.T., 1992a. Bifurcation and localization instabilities in cylindrical shells under bending. I. Experiments. Int. J. Solids Struct. 29 (9), 1117–1142.

Kyriakides, S., Ju, G.T., 1992b. Bifurcation and localization instabilities in cylindrical shells under bending. II Predictions. Int. J. Solids Struct. 29 (9), 1143–1171.

Kyriakides, S., Shaw, P.K., 1982. Response and stability of elastoplastic circular pipes under combined bending and external pressure. Int. J. Solids Struct. 18 (11), 957–973.

Kyriakides, S., Shaw, P.K., 1987. Inelastic buckling of tubes under cyclic loads. ASME J. Press. Vessel Technol. 109, 169–178.

Pan, W.F., Chern, C.H., 1997. Endochronic description for viscoplastic behavior of materials under multiaxial loading. Int. J. Solids Struct. 34 (17), 2131–2159.

Pan, W.F., Fan, C.H., 1998. An experimental study on the effect of curvature-rate at preloading stage on subsequent creep or relaxation of thin-walled tubes under pure bending. JSME Int. J. Ser. A 41 (4), 525–531.

Pan, W.F., Her, Y.S., 1998. Viscoplastic collapse of thin-walled tubes under cyclic bending. ASME J. Engng. Mater. Tech. 120, 287–290.

Pan, W.F., Hsu, C.M., 1999. Viscoplastic analysis of thin-walled tubes under cyclic bending. Int. J. Struct. Engng. Mech. 7 (5), 457–471.

Pan, W.F., Leu, K.T., 1997. Endochronic analysis for viscoplastic collapse of a thin-walled tube under combined bending and external pressure. JSME Int. J. Ser. A 40 (2), 189–199.

Pan, W.F., Wang, T.R., Hsu, C.M., 1998. A curvature-ovalization measurement apparatus for circular tubes under cyclic bending. Experiment. Mech. 38 (2), 99–102.

Shaw, P.K., Kyriakides, S., 1985. Inelastic analysis of thin-walled tubes under cyclic bending. Int. J. Solids Struct. 21 (11), 1073–1110.